Continuous Latent Space Representations of Networks from Event Data

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SSC 2026

Joint work with **Jie Jian** *(Chicago) and Jiguo Cao (SFU)*

Estimating Latent Political Alignment

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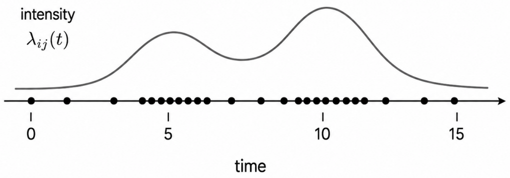
Estimating Latent Political Alignment

- Nodes in a network interact for a variety of reasons
- Interactions driven by latent characteristics
- Example is positive interactions (such as trade deals) between countries
  - These interactions can occur repeatedly, in continuous time
- Want to identify a flexible latent space representation of nodes given these interactions, relate to existing political theory

Relationships change over time

- Interactions occur repeatedly over time
- (Political) relationships evolve gradually
- Want to use underlying event data without aggregation
- Model repeated interactions in continuous time, learn smooth latent trajectories



The intensity driving events changes over time

Outline

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- Background on Latent Space models, extensions to dynamic models

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- Considerations in model fitting, computation
- Experimental results for simulated and real data

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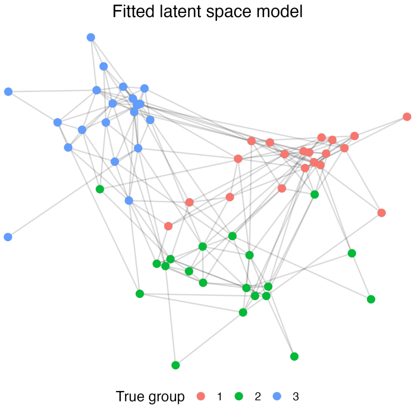
- Classical model proposed by Hoff, Raftery, and Handcock (2002)
- Probability of a binary edge p_{ij} given by

$$\text{logit}(p_{ij}) = \alpha + s(z_i, z_j),$$

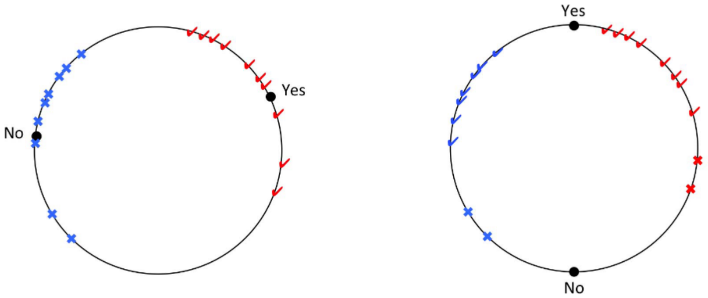
for some similarity metric $s(\cdot,\cdot)$ such as $s(z_i, z_j) = -||z_i - z_j||^2$ with $z_i \in \mathbb{R}^d$
- Closer nodes have a higher propensity to interact



A simple latent space model

Other Latent Spaces

- Latent space models have also been used with more flexible geometries such as hyperbolic space
- Circular latent space used for capturing voting patterns



(a) Partisan vote (b) Extremes votes together

A circular latent space (Yu and Rodríguez, 2021)

Extensions to Dynamic Networks

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- Extensions have (largely) considered multiple observations of networks in discrete time
- Sarkar and Moore (2005), Sewell and Chen (2015) and MacDonald, Levina, and Zhu (2025) some examples
- Information loss if aggregating underlying continuous time realisations

Continuous Time Latent Space Models

- Observe sequence of event times between node pair (i, j) given by $\mathcal{H}_{ij}(T) = \{t_{ij}^m\}_{m=1}^{M_{ij}}$, which we model with a point process with conditional intensity $\lambda_{ij}(t)$ with

$$L_{ij} = \left\{ \prod_{m=1}^{M_{ij}} \lambda_{ij}(t_{ij}^m) \right\} \exp\left\{ - \int_0^T \lambda_{ij}(s)\, ds \right\}.$$

- Across N nodes in a network observed in $[0, T]$, this gives likelihood

$$L = \prod_{i \neq j} L_{ij}.$$

- Parameterise $\lambda_{ij}(t)$ as a function of continuous time latent positions $z_i(t), z_j(t)$

Existing Work

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- Rastelli and Corneli (2023) considered continuous latent position model, with condition that $z_i(t)$ piece-wise linear with specified change points

$$\log \lambda_{ij}(t) = \beta - ||z_i(t) - z_j(t)||^2$$

- Gives closed form expressions for the likelihood, makes estimation (relatively) easy
  - Have to choose change points
- Romero et al. (2023) extend this model, fit using variational inference

- Incorporate node effects with

$$\log \lambda_{ij}(t) = \beta_i + \beta_j - ||z_i(t) - z_j(t)||^2$$

- Model $z_i(t)$ with a 2D B-spline basis representation

$$z_i(t) = \begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} = \begin{pmatrix} \sum_{k=1}^{K} c_{ik}^{(x)} \phi_k(t) \\ \sum_{k=1}^{K} c_{ik}^{(y)} \phi_k(t) \end{pmatrix}$$

- $\{\phi_k\}_{k=1}^{K}$ are B-spline basis functions and $c_{ik}^{(x)}, c_{ik}^{(y)}$ are node- and basis-specific coefficients

Model Specification, Penalties

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- Incorporate penalties to prevent excessive flexibility in this model
- Penalise both overall displacement of nodes and smoothness of node trajectories, giving overall

$$\text{objective} = \ell(c, \beta) - \lambda_1 \sum_i \int ||z_i'(t)||^2 dt - \lambda_2 \sum_i \int ||z_i''(t)||^2 dt$$

- In practice smoothness penalty more important

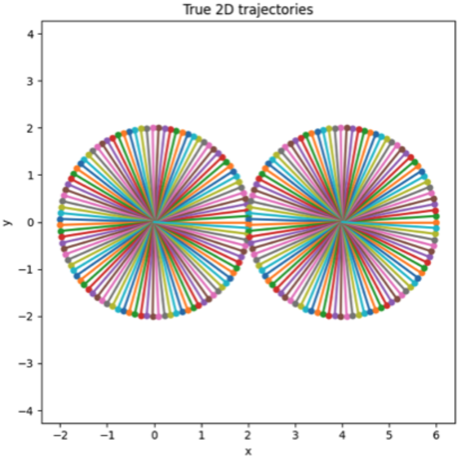
Initialization

- Flexible model which is hard to fit, good initialization important
- Fit static Poisson latent space to both halves of observation period, construct linear trajectory joining these two configurations
- Project this linear model onto a B-spline representation, use this as initial values for coefficients

Model Fitting

- Fix position of nodes to help with identifiability
- Computing objective expensive as number of nodes increases
- Use mini-batches to form stochastic gradient updates for the spline coefficients
- Update β_i with a fixed point algorithm, given current estimate of latent trajectories

Simulation Study

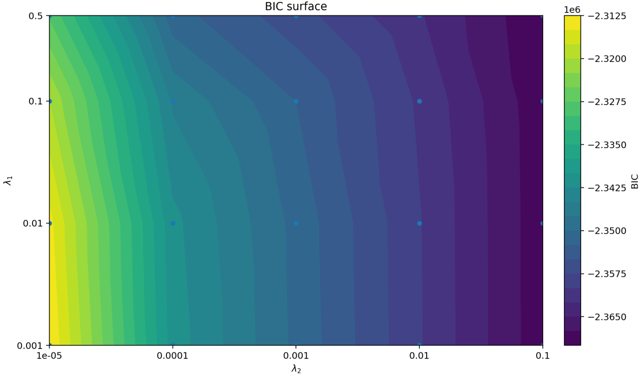


Simulated True Latent trajectories

- Simulate linear trajectories collapsing to two points
- Investigate impact of tuning parameters, recovery of positions

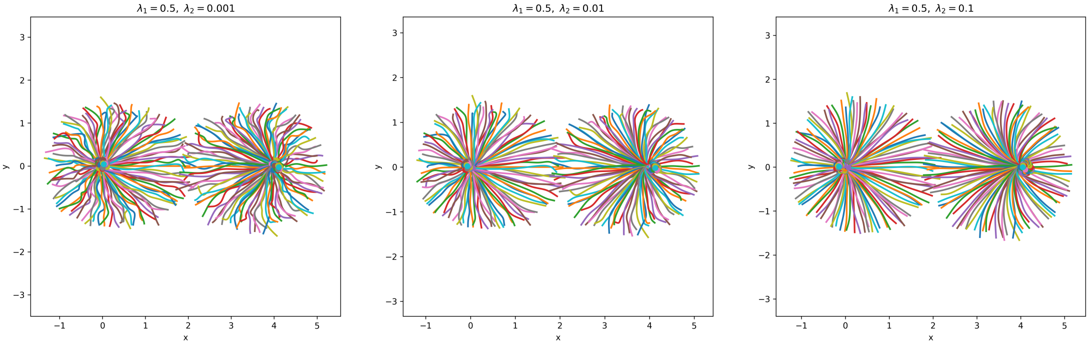
Choose Tuning Parameters with BIC

$$\text{BIC}(\lambda_1, \lambda_2) = -2\ell(\hat{c}, \hat{\beta}) + k_{\text{eff}}(\lambda_1, \lambda_2) \log(\text{total number of events})$$



Model selection is more sensitive to λ_2

Effect of the smoothing parameter

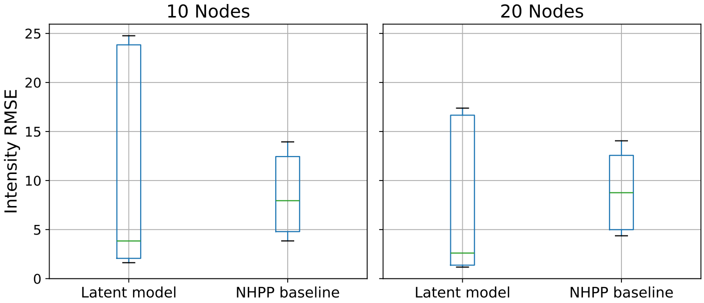


$\lambda_2 = 0.001$ $\lambda_2 = 0.01$ $\lambda_2 = 0.1$

Larger λ_2 produces progressively smoother trajectories while preserving the main shape.

Recovery in Simulations

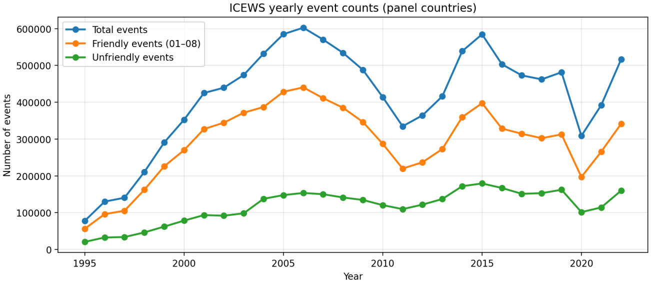
- Simulate from true B-spline trajectories
- Recover the underlying intensities well, comparable to flexible baseline (no latent structure)
- Good estimation of β_i



Intensity RMSE decreases as number of nodes increases

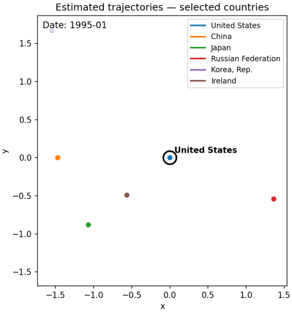
Real Data

- ICEWS Dataset provides timestamped (friendly) interactions between 60 countries
- Fit our proposed continuous time latent space model
- Examine latent trajectories of selected countries

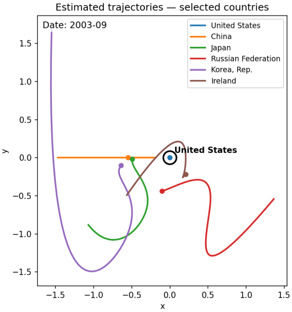


Yearly political interactions

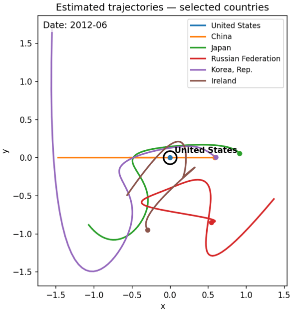
Real Data Results, 6 Countries



1995



2003



2012

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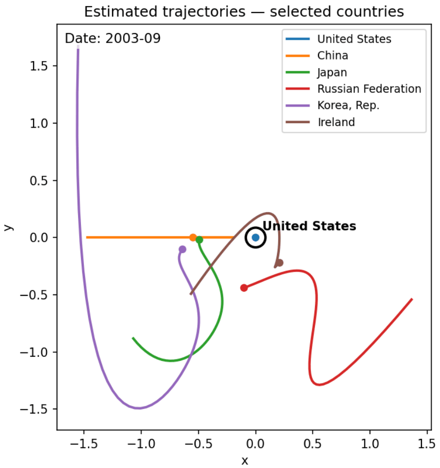
- Constrain one node fixed (US), one on axis (China)
- Ireland remains close to US
- Korea and Japan have similar estimated trajectories



2003

Country Baselines

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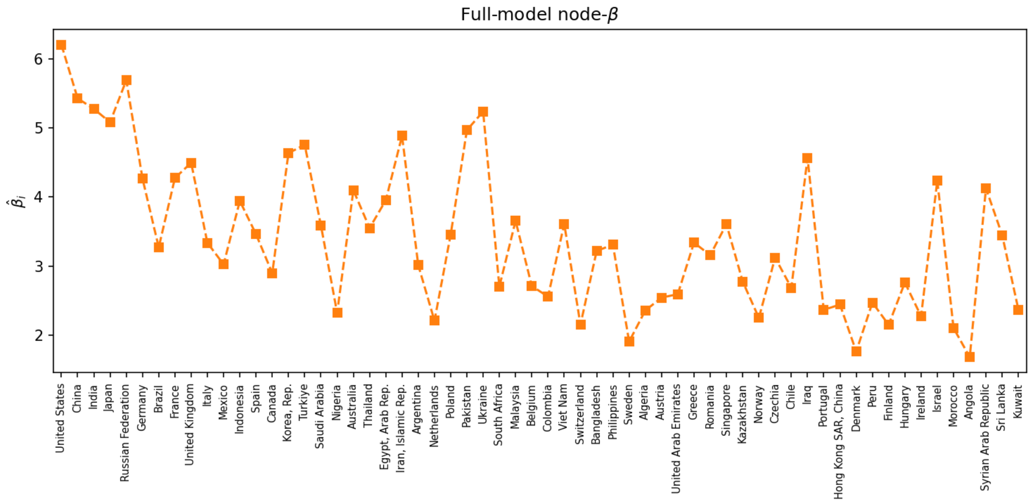

Larger estimates correspond to more “active” countries

Next Steps

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- Current identifiability constraints work reasonably well in practice, but need further investigation
- Initialization procedure is important in estimated trajectories
- Could incorporate self-excitation, covariates also

Summary

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- Proposed a flexible continuous-time latent space model for network event data
- B-spline trajectories provide smooth, interpretable evolution of latent positions
- Initialization and smoothing are important for stable recovery in simulations
- ICEWS application illustrates model can summarize dynamic patterns of (friendly) international interactions

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Thank you