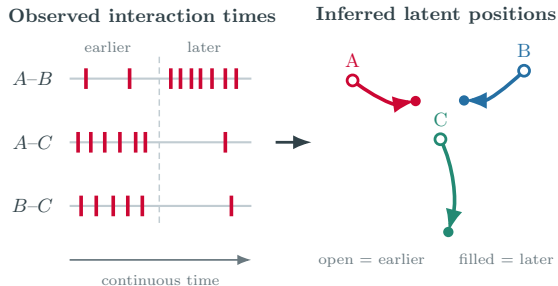


# Continuous Latent Space Representations of Networks from Event Data

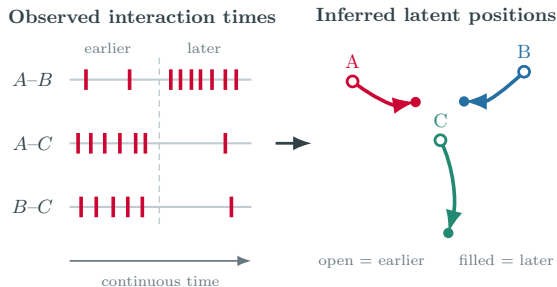
Owen G. Ward — Department of Statistics and Actuarial Science, SFU  
Joint work with Jie Jian (Chicago) and Jiguo Cao (SFU)

- Countries interact repeatedly over time, for example through trade deals.



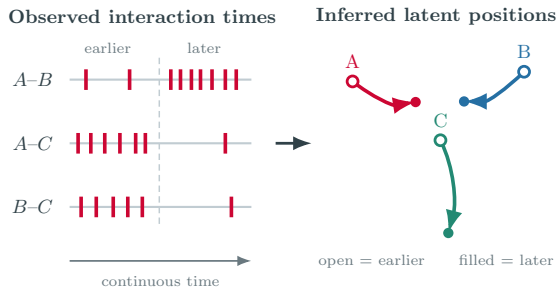
Number of events depends on positions in latent space

- Countries interact repeatedly over time, for example through trade deals.
- Hidden political alignments help drive those interactions.



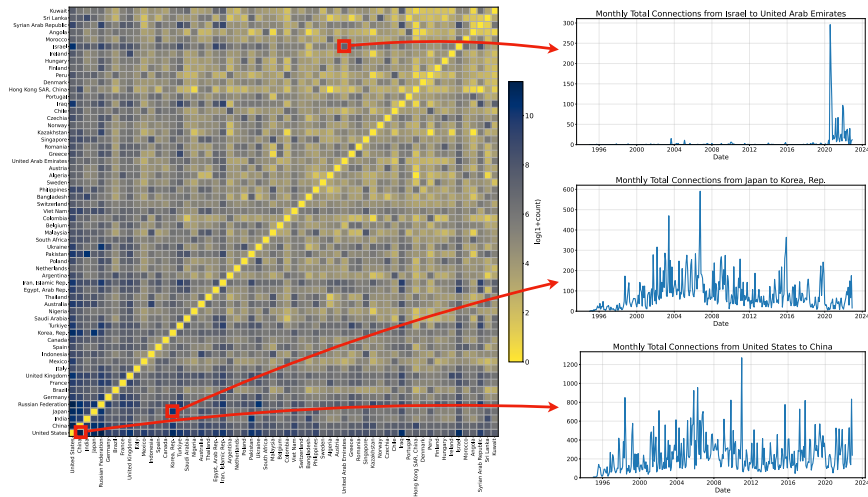
Number of events depends on positions in latent space

- Countries interact repeatedly over time, for example through trade deals.
- Hidden political alignments help drive those interactions.
- We infer how countries' positions change from the timing of interactions.



Number of events depends on positions in latent space

# Relationships change over time



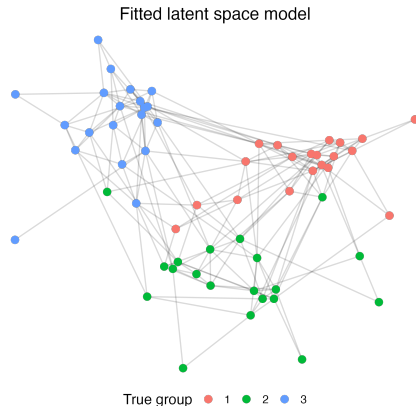
Aggregating total events hides temporal structure

- Classical model proposed by Hoff, Raftery, and Handcock (2002)
- Probability of a binary edge  $p_{ij}$  given by

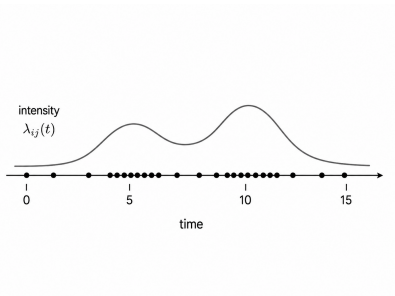
$$\text{logit}(p_{ij}) = \alpha + s(z_i, z_j),$$

for some similarity metric  $s(\cdot, \cdot)$  such as  $s(z_i, z_j) = -\|z_i - z_j\|^2$  with  $z_i \in \mathbb{R}^d$

- Closer nodes have a higher propensity to interact



A simple latent space model



The intensity driving events changes over time

- Extensions have (largely) considered multiple observations of networks in discrete time
- Sarkar and Moore (2005), Sewell and Chen (2015), and MacDonald, Levina, and Zhu (2025) are examples
- Information loss if aggregating underlying continuous time realisations

- Want to model exact event times, not discrete time aggregation
- Observe sequence of events between node pair  $(i, j)$  given by  $\mathcal{H}_{ij}(T) = \{t_{ij}^m\}_{m=1}^{M_{ij}}$ , which we model with a point process with conditional intensity  $\lambda_{ij}(t)$  with

$$L_{ij} = \left\{ \prod_{m=1}^{M_{ij}} \lambda_{ij}(t_{ij}^m) \right\} \exp \left\{ - \int_0^T \lambda_{ij}(s) ds \right\}, \quad L = \prod_{i \neq j} L_{ij}.$$

- Parameterise  $\lambda_{ij}(t)$  as a function of continuous time latent positions  $z_i(t), z_j(t)$

- Incorporate node effects, giving

$$\log \lambda_{ij}(t) = \beta_i + \beta_j - ||z_i(t) - z_j(t)||^2$$

- Model  $z_i(t)$  with a 2D B-spline basis representation

$$z_i(t) = \begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} = \begin{pmatrix} \sum_{k=1}^K c_{ik}^{(x)} \phi_k(t) \\ \sum_{k=1}^K c_{ik}^{(y)} \phi_k(t) \end{pmatrix}$$

- $\{\phi_k\}_{k=1}^K$  are B-spline basis functions and  $c_{ik}^{(x)}, c_{ik}^{(y)}$  are node- and basis-specific coefficients
- Previous work forced  $z_i(t)$  to be piece-wise linear

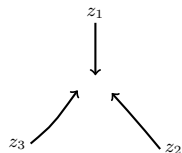
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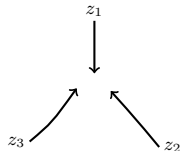
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- Choosing a good initialization important

Trajectory estimation for three converging nodes

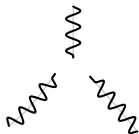


(a) True reference configuration

Trajectory estimation for three converging nodes

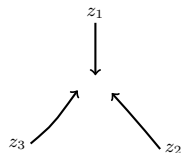


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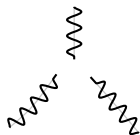


(b) Unpenalized splines:  
overfitting

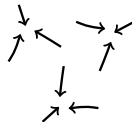
## Trajectory estimation for three converging nodes



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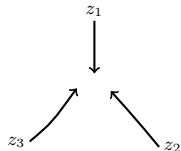


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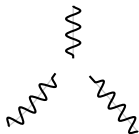


(c) Without anchors: rigid-motion ambiguity

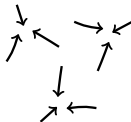
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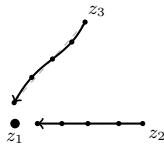
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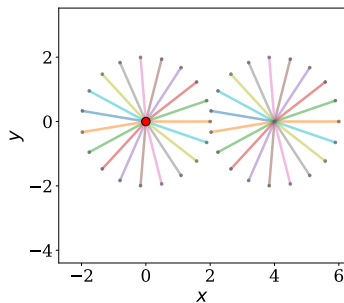
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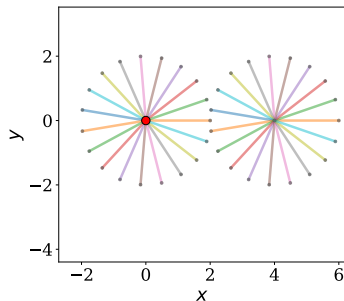


(d) Anchored, penalized splines: smooth recovery

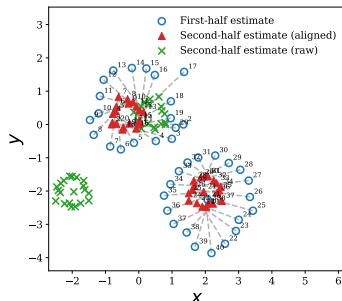


(a) True trajectories

Fit each half of the observation period separately, align the positions, then connect them to initialize the trajectories.

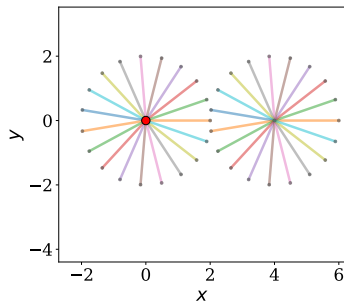


(a) True trajectories

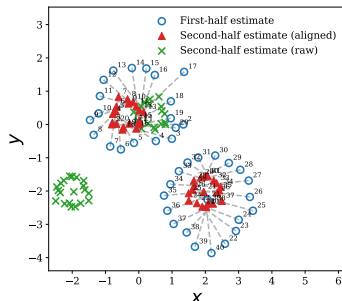


(b) Static fits in two time blocks

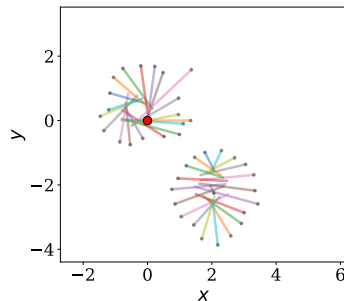
Fit each half of the observation period separately, align the positions, then connect them to initialize the trajectories.



(a) True trajectories



(b) Static fits in two time blocks



(c) Connected starting paths

Fit each half of the observation period separately, align the positions, then connect them to initialize the trajectories.

$$\log \lambda_{ij}(t) = \beta_i + \beta_j - \|z_i(t) - z_j(t)\|^2$$

Model  $z_i(t)$  with a 2D B-spline basis representation

$$z_i(t) = \begin{pmatrix} x_i(t) \\ y_i(t) \end{pmatrix} = \begin{pmatrix} \sum_{k=1}^K c_{ik}^{(x)} \phi_k(t) \\ \sum_{k=1}^K c_{ik}^{(y)} \phi_k(t) \end{pmatrix}$$

- Estimation of the spline coefficients uses stochastic gradient updates, closed form updates for  $\beta$

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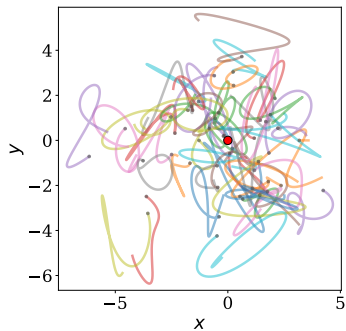
- Estimation of the spline coefficients uses stochastic gradient updates, closed form updates for  $\beta$
- Choose smoothing parameters  $(\rho_1, \rho_2)$  using BIC

$$\log \lambda_{ij}(t) = \beta_i + \beta_j - \|z_i(t) - z_j(t)\|^2$$

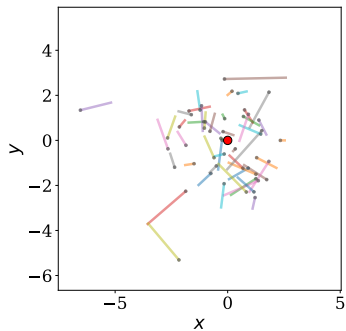
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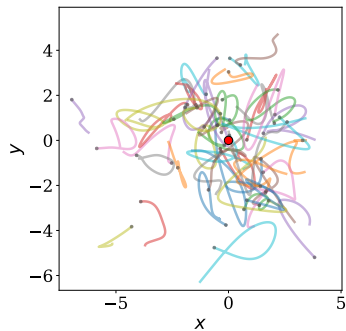
- Estimation of the spline coefficients uses stochastic gradient updates, closed form updates for  $\beta$
- Choose smoothing parameters  $(\rho_1, \rho_2)$  using BIC
- Can evaluate our model in terms of predictive performance, also model checking for point processes



True

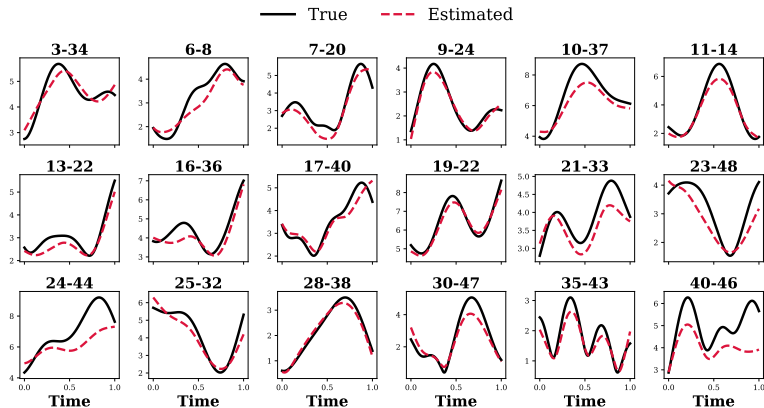


Initialization

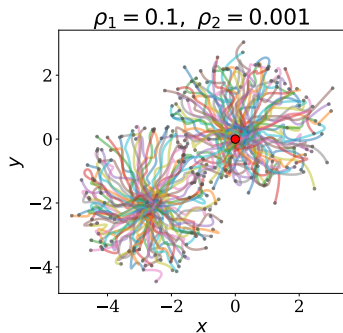


Estimation

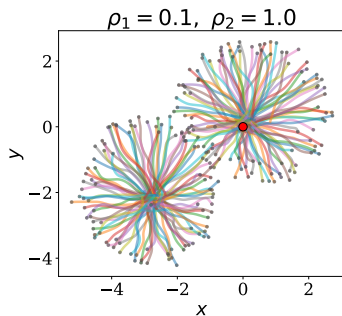
True, initial, and estimated trajectories in one simulation.



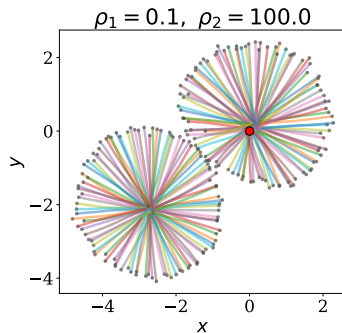
Randomly selected pairwise distances



$\rho_2 = 0.001$



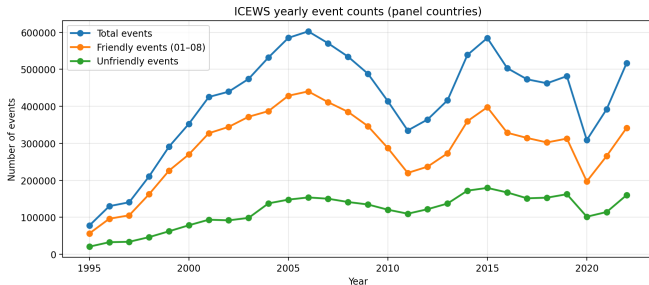
$\rho_2 = 1$



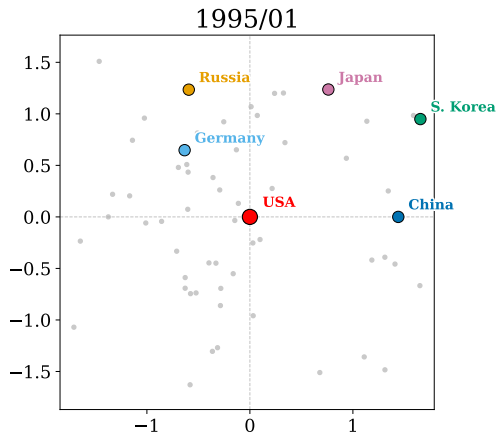
$\rho_2 = 100$

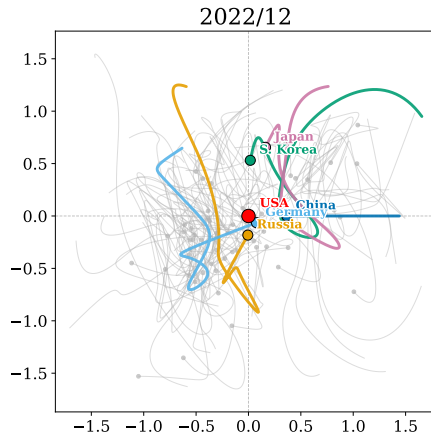
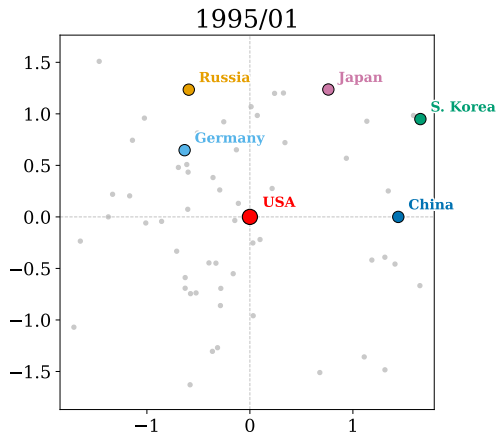
Different simulation: stronger smoothing produces straighter paths while preserving the two-cluster structure.

- ICEWS provides timestamped interactions between countries, 1995-2022
- Consider “friendly” interactions between 60 countries with largest GDP
- Fit our model, good predictive performance to comparable methods



Yearly political interactions





- Japan and Korea move together, general trend towards US

Largest and smallest estimated trajectory lengths ( $10^3$  units).

| Most mobile          |        | Least mobile               |        |
|----------------------|--------|----------------------------|--------|
| Country              | Length | Country                    | Length |
| Ukraine              | 7.01   | United States <sup>†</sup> | 0.00   |
| Iraq                 | 5.57   | Canada                     | 1.46   |
| Poland               | 4.92   | United Kingdom             | 1.62   |
| Pakistan             | 4.75   | Netherlands                | 1.67   |
| Sri Lanka            | 4.59   | Ireland                    | 1.73   |
| Brazil               | 4.28   | China <sup>‡</sup>         | 1.78   |
| Syrian Arab Republic | 4.17   | Switzerland                | 1.92   |
| Iran, Islamic Rep.   | 4.09   | Italy                      | 1.95   |
| Saudi Arabia         | 4.01   | France                     | 2.03   |
| Korea, Rep.          | 4.01   | Israel                     | 2.08   |

<sup>†</sup>U.S. fixed at origin; <sup>‡</sup>China constrained to horizontal axis. These anchor lengths are not directly comparable with the others.

- Event times reveal smooth changes in network relationships
- Node effects separate activity from relational proximity
- ICEWS trajectories summarize changing patterns of cooperation

Thank you

Paper: [arxiv.org/abs/2609.08813](https://arxiv.org/abs/2609.08813)

